PRACTICE EXAM (MID-YEAR) SOLUTIONS

Year 12 Mathematics Methods Exam 2

Part 1

Multiple-choice questions

1 C
2 E
3 E
4 A
5 B
6 C
7 C
8 B
9 E
10 A
11 C
12 D
13 A
14 B
15 E
16 C
17 D
18 A
19 C
20 A

Part 2

Long-answer questions

Question 1

a i $P(\text{no red lights}) = 0.7 \times 0.6 \times 0.5 \times 0.4 = 0.084$

[2 marks]

ii $P(R_2 \cap R_3 \cap R_4 / R_1) = 0.4 \times 0.5 \times 0.6 = 0.12$

[2 marks]

b i $g(x) = e^{-x} + \dfrac{2}{\sqrt{x}} - \sin(4x)$

$g'(x) = -1 \times e^{-x} - x^{-3/2} - 4\cos(4x)$

$= -\dfrac{1}{e^x} - \dfrac{1}{\sqrt{x^3}} - 4\cos(4x)$

$g'(\pi) = -\dfrac{1}{e^\pi} - \dfrac{1}{\sqrt{\pi^3}} - 4\cos(4\pi)$

$g'(\pi) = -\dfrac{1}{e^\pi} - \dfrac{1}{\sqrt{\pi^3}} - 4$

[2 marks]

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ii $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\cos 2x - 1)dx = \left[\frac{\sin(2x)}{2} - x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$$= \left(\frac{\sin(\pi)}{2} - \frac{\pi}{2}\right) - \left(\frac{\sin\left(\frac{2\pi}{3}\right)}{2} - \frac{\pi}{3}\right)$$

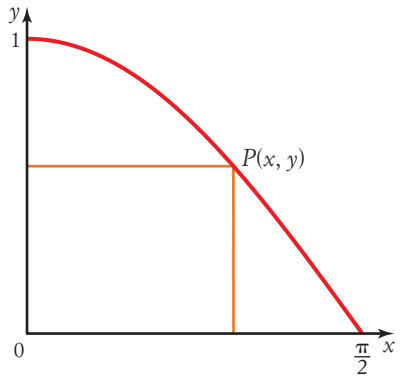
$$= -\frac{\pi}{2} - \frac{\sqrt{3}}{4} + \frac{\pi}{3}$$

$$= -\frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

[2 marks]

Question 2

a $y = \cos(x)$ for $0 \leq x \leq \frac{\pi}{2}$.



Area $_{\text{rectangle}} = x \times y = x\cos(x)$

Max. area when $\frac{dA}{dx} = 0$ and $\frac{d^2A}{dx^2} < 0$

$\frac{dA}{dx} = 1 \times \cos(x) - x \times \sin(x)$

$\frac{d^2A}{dx^2} = -\sin(x) - [1 \times \sin(x) + x \times \cos(x)]$

$\frac{d^2A}{dx^2} = -x\cos(x) - 2\sin(x)$

If $\frac{dA}{dx} = 0$,

$\cos x = x\sin(x)$

$x = 0.86033$ (radians)

$\frac{d^2A}{dx^2} = -0.86033\cos(0.86033) - 2\sin(0.86033) < 0$, therefore max. dimensions are $x \times \cos(x)$

i.e. 0.86×0.65

[5 marks]

b Area $= 3\int_{-\frac{\pi}{2}}^{\pi} |\cos(x)|\, dx$

$= 3\int_{0}^{\frac{\pi}{2}} \cos(x)\, dx$

$= 3\left[\sin(x)\right]_{0}^{\frac{\pi}{2}}$

$= 3\left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right]$

$= 3 \text{ units}^2$

[3 marks]

c $x = \cos(t)$

i $v = -\sin(t)$

max. v for $0 = t = 2\pi$ is 1

[2 marks]

ii $a = -\cos(t)$

At $x = \frac{1}{\sqrt{2}}$

$\therefore a = -\frac{1}{\sqrt{2}}$

[2 marks]

Question 3

Two normal six-sided dice are rolled.

a Outcome: 6,6 $\overline{6,6}$

Probability: $\frac{1}{36}$ $\frac{35}{36}$

$E(X) = np = 50 \times \frac{1}{36} = 1.39$

$S(x) = \sqrt{npq} = \sqrt{50 \times \frac{1}{36} \times \frac{35}{36}} = 1.16$

[3 marks]

b **i** Difference table:

0	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

[2 marks]

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ii

x	0	1	2	3	4	5
$P(X = x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

[2 marks]

iii $E(X) = 0 \times \frac{6}{36} + 1 \times \frac{10}{36} + 2 \times \frac{8}{36} + 3 \times \frac{6}{36} + 4 \times \frac{4}{36} + 5 \times \frac{2}{36}$

$= 1.94$

[2 marks]

iv The most likely difference is 1.

[1 mark]

Question 4

a Got 15 correct, want 2 more from 5 to get 17.

$n = 5, p = 0.2, P(x = 2) = 0.2048$

[3 marks]

b Got 7 correct, want 3 or more from 13 to pass.

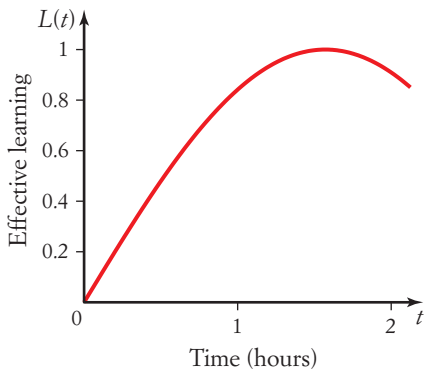
$n = 13, p = 0.2, P(x \geq 3) = 1 - P(x \leq 2)$

$= 1 - 0.5017$

$= 0.4983$

[3 marks]

c $L(t) = \sin(t)$ for $0 \leq t \leq 2$



$\frac{dL}{dt} = \cos(t)$

$\frac{dL}{dt} \approx \frac{\delta L}{\delta t}$

$\delta L \approx \delta t \times \frac{dL}{dt}$

At $t = 1$,

$\delta L \approx 0.25 \times \cos(1) = 0.135$

The percentage increase in effective learning

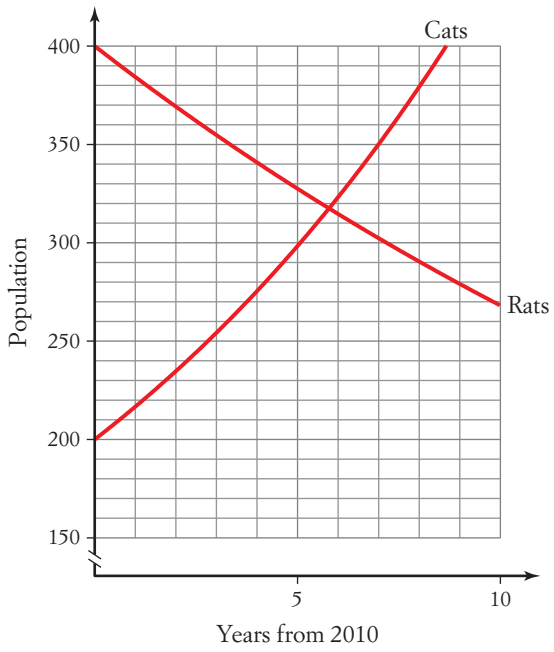
$= \dfrac{\delta L}{L} \times 100\% = \dfrac{0.135}{\sin(1)} \times 100$

$= 16\%$

[4 marks]

Question 5

a



[3 marks]

b $P_r = 400e^{-0.04t}$

$100 = 400e^{-0.04t}$

$t = 34.66$

2044.66, i.e. in the year 2045

[2 marks]

c i $P_{wc} = 200e^{0.08t}$

$\dfrac{dP_{wc}}{dt} = 200e^{0.08t} \times 0.08$

$\dfrac{dP_{wc}}{dt} = 16e^{0.08t}$

i.e. rate of growth of cat population is $16e^{0.08t}$

$P_r = 400e^{-0.04t}$

$\frac{dP_r}{dt} = 400e^{0.04t} \times (-0.04)$

$\frac{dP_r}{dt} = -16e^{0.04t}$

i.e. rate of growth of rat population is $-16e^{0.04t}$

[2 marks]

ii The population of wild cats is increasing at the fastest rate because it is a positive rate. The rate of increase of the rat population is negative.

[2 marks]

d $200e^{0.08t} = 400e^{-0.04t}$

$t = 5.78$

i.e. 2016; that is, the rats would die at a faster rate in the year 2016.

[2 marks]

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